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Gas equation During an Adiabatic process:-

Let us consider 1 mole of the working substance like ideal gas perfectly insulated from the surrounding. Let the external work done by the gas be dW.

Applying the first law of thermodynamics,

$$dH = dU + dW$$

But $dH = 0$

and $dW = P.dV$

where P is the pressure of the gas and dV is the change in volume.

$$\therefore \quad 0 = dU + \frac{P.dV}{J} \quad \text{—— (i)}$$

As the external work is done by the gas at the cost of its internal energy, there is fall in tempr. by dT.

$$dU = 1 \times C_v.dT$$

$$C_v.dT + \frac{P.dV}{J} = 0 \quad \text{—— (ii)}$$

For an ideal gas,

$$PV = RT \quad \text{—— (iii)}$$

differentiating both side,

$$P.dV + V.dP = R.dT$$

$$\therefore \quad dT = \frac{PdV + VdP}{R}$$

Put the value dT in eqn. (ii), we get,

$$C_v\left[\frac{PdV + VdP}{R}\right] + \frac{PdV}{J} = 0$$

$$C_v[PdV + VdP] + \frac{R\,PdV}{J} = 0$$

But $\frac{R}{J} = C_p - C_v$

$$\therefore \quad C_v\,PdV + C_v.VdP + PdV(C_p - C_v) = 0$$

$$\cancel{C_v\,PdV} + C_v.VdP + C_p\,PdV - \cancel{C_v\,PdV} = 0$$

$$C_p P dV + C_v V dP = 0$$

Dividing by $C_v PV$

$$\frac{C_p P dV}{C_v PV} + \frac{\cancel{C_v}\,\cancel{V} dP}{\cancel{C_v} P\cancel{V}} = 0$$

$$\frac{C_p}{C_v} \cdot \frac{dV}{V} + \frac{dP}{P} = 0 \quad \because \frac{C_p}{C_v} = \gamma$$

$$\therefore \frac{dP}{P} + \gamma \frac{dV}{V} = 0$$

Integrating both side

$$\int \frac{dP}{P} + \int \gamma \frac{dV}{V} = \int 0$$

$$\log P + \gamma \log V = \text{Constant}$$

$$\log PV^{\gamma} = \text{Constant}$$

$$\therefore PV^{\gamma} = \text{Constant} \quad \text{--- (iv)}$$

This is equation of connecting pressure and volume during an adibetic process.

taking $PV = RT$

$$V = \frac{RT}{P}$$

From eqn (iv), $P\left[\frac{RT}{P}\right]^{\gamma} = \text{Const.}$

$$\frac{P^1 R^{\gamma} T^{\gamma}}{P^{\gamma}} = \text{Const.}$$

$$P^{1-\gamma} \cdot T^{\gamma} = \frac{\text{Const}}{R^{\gamma}} = \text{Const.}$$

$$\therefore T^{\gamma} \cdot P^{1-\gamma} = \text{Const} \quad \text{--- (V)}$$

Also $P = \frac{RT}{V}$

From eqn. (iv), $\frac{RT}{V^1} \cdot V^{\gamma} = \text{Const.}$

$$V^{\gamma-1} T = \frac{\text{Const}}{R} = \text{Const}$$

$$\therefore T \cdot V^{\gamma-1} = \text{Const.} \quad \text{--- (vi)}$$